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Applying the Augmented *Q*-Model for Investment to US Aggregate Data

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ABSTRACT
The standard Q-model for investment, augmented to include a liquidity variable (cash flow or retained profits) as a proxy for the nexus between investment and financial constraints, is usually applied to firm-level panel data. A relatively small number of studies apply the model to aggregate data as well. To our knowledge, however, two important econometric issues have not received proper attention: the possible nonstationarity of Q and the presence of structural breaks. Using United States (US) aggregate quarterly data, 1947:1-2022:2, we find evidence that (1) Q behaves as an I(1) variable, whereas the other variables of the equation behave as I(0), and (2) there is a structural break around 1963:1. Accordingly, we use the "bounds testing" approach and allow the regression coefficients to change from 1963:1 onward. As a result, the coefficient of Q, which is statistically significant if the break is ignored, becomes insignificant. This finding coupled with evidence for omitted nonlinearities suggests that the model may be inadequate.

KEYWORDS
Investment, financial constraints, Q-model, structural breaks, bounds test

1. Introduction

Aggregate investment is crucial for economic growth and plays a major role in the business cycle, as its fluctuations dominate the cycle ([13, p. 377]; [8, p. 228]). Thus, it is important to have an empirically reliable model for aggregate investment for forecasting aggregate fluctuations as well as for enacting stabilization and growth policies ([3, pp. 163, 185]). One problem is how to model financial (or credit) constraints. In 1993, Chirinko [11] wrote that "the investment literature has been schizophrenic concerning the role of financial structure and liquidity constraints" (p. 1902). Today, modeling the nexus between investment and credit constraints is still problematic.

The standard empirical model for testing this nexus is Tobin's Q-model augmented to include a liquidity variable, l_t (cash flow or retained profits), as a proxy for financial constraints. It is a "commonly used investment function" ([8, p. 206]) after the criticism of the original Q-model on both theoretical and empirical grounds ([10, p. 70]; [9, p. 278]) and has been predominantly applied to firm-level panel data by employing

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various sample-splitting criteria (firm size, bond rating, etc.) to classify firms as financially constrained or unconstrained. The severity of credit constraints is measured by the coefficient of l_t, which is expected to be positive and statistically significant ([11, pp. 1902-1903]; [17, p. 543]; [24, p. 204]). The fraction of financially constrained firms in the economy, a crucial policy parameter, denoted as λ, can be estimated from firm-level data using the standard statistical inference on proportions (see section 3). With firm-level data, λ is estimated more precisely than with aggregate data, because of the limited variation in the latter, but is econometrically over-identified, in that its estimates differ significantly, depending on which sample-splitting criterion is used.

A number of studies that apply the Q-model to aggregate data try to overcome its empirical failures by using alternative definitions of the variables or alternative sample periods ([10]; [14]; [33]; [32]; [1]). To our knowledge, two important econometric issues have not received proper attention in the literature: the possible nonstationarity of Q and structural instability.

This paper stresses these two issues. We use United States (US) aggregate quarterly data, 1947:1-2022:2, and find evidence that Q behaves as an I(1) variable, whereas the other variables in the equation behave as I(0), so the appropriate estimation method is the "bounds testing" approach. In addition, we find evidence for a few structural breaks, the most important of which occurred around 1963:1, possibly because of the investment tax credit introduced in October 1962 as well as the financial liberalization and the supply-side fiscal policies enacted in the early 1980s, particularly the 1981 and 1982 tax acts aimed to boost investment ([30]; [34, p. 5]). Accordingly, we allow both the regression intercept and the slopes to change from 1963:1 onward. As a result, the coefficient of Q, which is statistically significant if the break is ignored, becomes insignificant. This finding coupled with evidence for omitted nonlinearities suggests that the model may be inadequate, a conclusion reached because of our econometric modeling strategy, which stands in sharp contrast with that used in the existing literature.

More specifically, we derive the estimating equation in the same way as in the aggregate consumption literature that abandons the representative-agent assumption and combines permanent-income (unconstrained) and "rule-of-thumb" (credit constrained) behavior ([4]; [5]; [6]; [21]). In this setting, we show that, under certain assumptions, λ is exactly identified. We estimate λ as 0.49 for the post-1963:1 period, which implies that 49 percent of the US firms may be giving up attractive investment opportunities because of credit constraints.

Section 2 derives the model, section 3 estimates it and compares the estimates of λ with those obtained from firm-level data, and section 4 concludes.

2. The model

Consider two types of firms: type 1 firms, which face no credit constraints and determine their investment spending in accordance with the original Q-model; and type 2 firms, which are financially constrained and determine their investment in accordance with the augmented Q-model ([16]). Following Hatzinikolaou [21, pp. 294-295], let N_t denote the total number of firms in the economy at time t, N_{1t} of which face no credit constraints, and N_{2t} are financially constrained, so $N_t = N_{1t} + N_{2t}$. Define $\lambda = N_{2t}/N_t$, the proportion of financially constrained firms, and $1 - \lambda = N_{1t}/N_t$, the proportion of unconstrained firms. Finally, let I'_{it} denote total investment of type i firms, $i = 1, 2$, and let I'_t denote the published series for aggregate investment, so $I'_t = I'_{1t} + I'_{2t}$. Investment per firm is $I_t = I'_t/N_t$ and $I_{it} = I'_{it}/N_{it}$. Divide $I'_t = I'_{1t} + I'_{2t}$ by last period's capital stock, K_{t-1}, and multiply and divide the term on the left by N_t, the first term on the right by N_{1t}/N_t, and the second term by N_{2t}/N_t. We get the identity

$$IK_t = (1 - \lambda)\, IK_{1t} + \lambda\, IK_{2t}, \tag{1}$$

where $IK_t = (I'_t/N_t)/(K_{t-1}/N_t) = I'_t/K_{t-1}$ and $IK_{it} = (I'_{it}/N_{it})/(K_{t-1}/N_t)$, so "investment" here is the ratio of investment per firm to last period's capital stock per firm. In estimation, we use only the variable IK_t, which can be constructed from the series I'_t and K_{t-1}, without knowledge of N_t.

The original Q-model modifies the neoclassical optimization model for investment by taking explicitly into account adjustment costs ([22, p. 214]). The variable Q is a firm's ratio of the market value of an *additional* unit of capital to its replacement cost, and is called marginal Q. If Q were observable, it could serve as a "sufficient statistic" summarizing the firm's investment opportunities ([24, p. 200]), in that the optimal rate of investment is determined by Q, adjusted for tax parameters. The equilibrium value of Q is 1, in which case the firm is worth what it costs to replace its assets; if $Q > 1$, the firm is overvalued, and can finance its investment by issuing a relatively small number of new shares ([8, p. 280]); whereas if $Q < 1$, the firm is undervalued, and financing investment by issuing new shares is relatively expensive. Because marginal Q is not observable, the Q-model is made operational by using average Q as a proxy, the ratio of the market value of *existing* capital to its replacement cost. Under certain assumptions about technology and market conditions, marginal Q equals average Q ([22]). The augmented Q-model is

$$IK_t = \mu_0 + \mu_1 Q_t + \mu_2 l_t + u_t, \tag{2}$$

where $l_t = RCP_t/K_{t-1}$ and RCP_t = retained corporate profits.[1] The preceding discussion suggests that $\mu_1 > 0$. In section 1, we saw that a positive and statistically significant value of μ_2 is usually taken as a measure of the extent of credit constraints (see, e.g., [11, pp. 1902-1903]; [24, p. 204]; [12]; [29]). We now derive Equation (2) so that we can interpret the coefficient μ_2 precisely.

Since the original Q-model assumes perfectly competitive input and output markets, we take the investment function of a type 1 firm to be (see [16, p. 165])

$$IK_{1t} = \mu_{10} + \mu_{11} Q_t + u_{1t}, \quad \mu_{11} > 0 \tag{3}$$

whereas that of a type 2 firm to be the augmented Q-model,

$$IK_{2t} = \mu_{20} + \mu_{21} Q_t + \mu_{22} l_{2t} + u_{2t}, \tag{4}$$

[1] Our initial set of explanatory variables in (2) included CP_t/K_{t-1} = corporate profits as a ratio to last period's capital stock, CF_t/K_{t-1} = cash flow as a ratio to last period's capital, and p_t = price of investment goods. Our empirical analysis suggested, however, that, in the presence of Q, CP_t/K_{t-1} and p_t have no explanatory power for investment and must be dropped. It also suggested that either CF_t/K_{t-1} or RCP_t/K_{t-1} can be included in the regression, but not both. We have decided to include the latter, as it produces somewhat better results.

where $l_{2t} = RCP_{2t}/K_{t-1}$ = retained corporate profits per unit of last period's capital of a type 2 firm. In (3) and (4), we assume that investment of both types of firms depends on the average Q_t for the whole market. This assumption may not be unreasonable, as Q_t reflects investment opportunities, which firms of both types can see equally well. The two types of firms may react differently to changes in Q_t, however. Consider, for example, two firms, one financially constrained and the other unconstrained. A *ceteris paribus* increase in Q_t may cause the unconstrained firm to invest more than the constrained one, because it can issue external debt, implying that $\mu_{11} > \mu_{21}$.[2]

Multiply (3) by $(1-\lambda)$ and (4) by λ; add the two resulting equations; use identity (1); and assume that total retained profits of type 2 firms are a fraction λ of the aggregate figure, i.e., $RCP_{2t} = \lambda RCP_t$. This assumption is similar to that used in the aggregate consumption literature, which says that the "rule of thumb" consumers receive a fraction λ of the aggregate income, i.e., $Y_{2t} = \lambda Y_t$ ([4, p. 188]; [5, p. 266]; [6, p. 728]; [21, p. 295]). The result is our Equation (2), where

$$\mu_0 = (1-\lambda)\,\mu_{10} + \lambda\mu_{20},\ \mu_1 = (1-\lambda)\mu_{11} + \lambda\mu_{21},\ \ \mu_2 = \lambda^2\mu_{22}, \qquad (5)$$

$$u_t = (1-\lambda)u_{1t} + \lambda u_{2t}.$$

The estimation of Equation (2) will yield estimates of the parameters μ_0, μ_1, and μ_2 and of their variances. From the definitions in (5), we see that we can identify λ by imposing the restriction $\mu_{22} = 1$, in which case $\mu_2 = \lambda^2$. We adopt this restriction, as it is intuitively not unreasonable to assume that the coefficient of retained corporate profits in (4) is unity, because it is likely "for a firm with good investment opportunities, that at time t or $t+1$ retentions are *all* used up (dividends are zero)" ([25, p. 372], our italics). This restriction allows us to estimate λ using the formula $\hat{\lambda} = \sqrt{\hat{\mu}_2}$.

3. Application to US aggregate data

We estimate Equation (2) using US aggregate quarterly data, 1947:1-2022:2, obtained from the US Department of Commerce, Bureau of Economic Analysis (BEA), National Income and Product Accounts (NIPA), and the Federal Reserve Bank of St. Louis (FRED). All the flow variables listed below are measured in b. $ and are seasonally adjusted (s.a.). We use the following variables: Nominal gross domestic product (*GDP*, BEA, Table 1.1.5); Private gross fixed nonresidential investment (*PGFNRI*, BEA, Table 5.3.5); $p_t = PI_t \times 100/PGDP_t$, where PI_t = price of private fixed nonresidential investment, and $PGDP_t$ = *GDP* deflator (BEA, Table 1.1.4, 2012 = 100); Real *GDP*, $Y_t = GDP_t \times 100/PGDP_t$; CP_t = corporate profits with inventory valuation adjustment and capital consumption adjustment (BEA, Table 1.12); RCP_t = retained corporate profits (BEA, Table 5.1); CF_t = net cash flow with inventory valuation adjustment (BEA, Table 1.12); Q_t = Tobin's Q, the ratio of the market value of equities outstanding to net worth of nonfinancial corporate businesses (Federal Reserve Bank of St. Louis);[3] K_t = capital stock (see Appendix A); $D63_t$, a dummy

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[2]If we replace Q_t with Q_{1t} in (3) and with Q_{2t} in (4) and assume that $Q_{it} = \beta_i Q_t + \varepsilon_{it}$, $i = 1, 2$, where β_i is a parameter and ε_{it} is a zero mean identically independently distributed (i.i.d.) random error, the definitions of μ_1 and u_t in (5) will become $\mu_1 = (1-\lambda)\beta_1\mu_{11} + \lambda\beta_2\mu_{21}$ and $u_t = (1-\lambda)(\mu_{11}\varepsilon_{1t} + u_{1t}) + \lambda(\mu_{21}\varepsilon_{2t} + u_{2t})$. These changes are not of interest here, however, because we focus on the coefficient of l_t.

[3]We are grateful to the FRED team, who kindly sent us the following link to retrieve this series for Tobin's Q: `https://fred.stlouisfed.org/graph/?g=YpxI`. For each of the years 1947:1-1950:4, only the figure for the fourth quarter is available; we used that figure for the other quarters of that year as well.

variable that takes on the value of 1 for all the quarters from 1963:1 onward and 0 otherwise, to capture possible effects of the investment tax credit policy introduced in October 1962 as well as those of the financial liberalization and the supply-side fiscal policies of the early 1980s, particularly the 1981 and 1982 tax acts aimed to boost investment spending ([30]; [34, p. 5]); $D96_t$, a dummy that equals 1 for the quarters 1996:2-2001:2 and 0 otherwise, to capture the euphoria of investment in the US of the late 1990s, reflecting the increase in productivity, due to advances in computer and communication technology ([23, pp. 2-3]); $D01_t$, a dummy that equals 1 for 2001:3-2004:1 and 0 otherwise, to capture the possible effect of the 9/11 event on investment; $D08_t$, a dummy that equals 1 for 2008:1-2010:1 and 0 otherwise, to capture the possible effect of the 2008 financial crisis. We determined the dates where these dummies equal 1 by simply inspecting the time-series graph for our series for investment, $IK_t = PGFNRI_t/K_{t-1}$ (see Figure 1). For example, the graph reflects a permanent increase in the level of IK_t, from an average value of 0.0942 during 1947:1-1962:4 to 0.1077 from 1963:1 onward, and an increase in its standard deviation from 0.00467 to 0.00784.

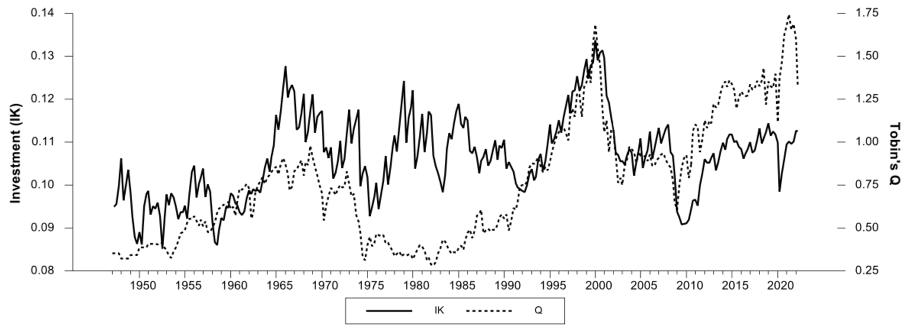

Figure 1. Our series for aggregate investment per unit of last period's capital stock (*IK*) and Tobin's *Q* from the Federal Reserve Bank of St. Louis, 1947:2 – 2022:2.

We begin our empirical analysis by testing for unit roots using the Phillips-Perron test (*PP*) and the Lee-Strazicich test ([26]and [27]), which allows for one or two structural breaks endogenously. Table 1 reports the results, which suggest that the series IK_t, RCP_t/K_{t-1} and CF_t/K_{t-1} can be taken as I(0), whereas the series Q_t as I(1). The unit-root hypothesis for the first differences of these variables, including that of Y_t, is strongly rejected.

Since Q_t is taken to be I(1), whereas all the other variables involved in Equation (2) as I(0), we must try to avoid the spurious-regression problem. Taking first differences in (2) and applying either ordinary least squares (OLS) or the generalized method of moments (GMM) yields a negative and insignificant coefficient on $\Delta Q_t = Q_t - Q_{t-1}$. An alternative series for Q_t, which we retrieved from the same source, [4] behaves also as I(1) and produces worse results, so we abandoned it. Under these circumstances, the most appropriate method for estimating (2) is the "bounds testing" (BT) approach

[4] `https://fred.stlouisfed.org/release/tables?rid=52&eid=810145` (Equities/net worth (percent), line 45 = line 42/line 39). This series is measured as a percentage, so we divided it by 100.

Table 1. Unit-root tests

Test / Series	PP_μ	PP_τ	*LS* one crash	*LS* two crashes	*LS* one break	*LS* two breaks	I(0) or I(1)?
IK_t	-3.78***	-4.08***	-5.65*** (2005:1)	-5.98*** (1964:4, 2005:1)	-6.10*** (1965:1)	-6.81*** (1963:2, 2003:4)	I(0)
Q_t	-1.51	-2.50	-2.74	-2.89	-3.23	-3.82	I(1)
$l_t = RCP_t/K_{t-1}$	-3.55***	-3.48**	-4.71*** (1980:1)	-5.35*** (1980:1, 2004:4)	-4.94*** (1980:1)	-5.52*** (1962:1, 1980:4)	I(0)
CF_t/K_{t-1}	-4.39***	-5.81***	-3.70** (1980:1)	-3.95** (1958:3, 1980:1)	-5.66*** (1968:4)	-5.88*** (1968:4, 2011:3)	I(0)
CP_t/K_{t-1}	-2.77*	-2.80	-3.44** (1974:1)	-3.92** (1969:1, 1980:1)	-4.99*** (1980:4)	-6.11** (1968:4, 1984:1)	I(0)
p_t	1.82	-4.34***	-1.01	-1.17	-2.25	-2.87	I(0)

Notes: (1) ***, **, * indicate significance at the 1-, 5-, and 10-percent level; (2) in the Phillips-Perron (*PP*) tests, the subscripts μ and τ indicate, respectively, "intercept-but-no-trend" and "intercept-plus-trend"; (3) in the Lee and Strazicich (*LS*, 2003, 2013) tests, the lag length was set equal to 4; (4) in the *LS* tests, the possible break dates of the levels are given in parentheses underneath the values of the test statistic; (5) the results for the first differences of the variables are not reported, as the unit-root hypothesis in the first differences is strongly rejected in every case; (6) the results have been generated by the econometric program *WinRATS* v. 10.0 (2019), which provides critical values for these tests.

of Pesaran, et al. [31], which is applicable regardless of whether the variables involved are I(0) or I(1), provided that there may be only one "levels relationship" involving the dependent variable (IK_t). Thus, we apply OLS (with robust standard errors to heteroscedasticity and to serial correlation in every case) to the following equation:[5]

$$\begin{aligned}\Delta(IK_t) = {} & a_0 + a_1 IK_{t-1} + a_2 Q_{t-1} + a_3 l_{t-1} + b_1 D63_{t-1} + b_2 D96_{t-1} \\ & + b_3 D08_{t-1} + b_4 (D63_{t-1} \times IK_{t-1}) + b_5 (D63_{t-1} \times Q_{t-1}) \\ & + b_6 (D63_{t-1} \times l_{t-1}) + \sum_{i=1}^{8} c_i \Delta(IK_{t-i}) + \sum_{i=1}^{8} d_i \Delta Q_{t-i} \\ & + \sum_{i=1}^{8} f_i \Delta l_{t-i} + \sum_{i=1}^{8} g_i \Delta Y_{t-i} + v_t. \end{aligned} \tag{6}$$

We start by estimating (6) after imposing the restrictions $b_1 = b_2 = b_3 = b_4 = b_5 = b_6 = 0$. This is our "restricted regression" in Part A of Table 2. It is the end result of many alternative configurations of variables and lagged differences, in an effort to achieve "the best" possible combination of the following four objectives: (1) reject the hypothesis $H_0 : a_1 = a_2 = a_3 = 0$ using the BT, thus establishing a "levels

[5]Pesaran, et al. ([31, footnote 17]) note that the asymptotic theory and the critical values for the "bounds test" are valid only to the extent that the fraction of periods in which the dummy variables are non-zero is close to zero. For $D96_t$ and $D08_t$, these fractions are 1.66% and 2.98%, respectively. For $D63_t$, this fraction is large, however, so we performed the test both with and without $D63_t$ in the estimating equation. As we report below, in both cases, the test rejects strongly the hypothesis $H_0 : a_1 = a_2 = a_3 = 0$ at a level way below 1-percent. Note also that when we included in the equation the dummy $D01_t$, it turned out to be statistically insignificant, so we dropped it.

relationship"; (2) obtain "correctly" signed and statistically significant coefficients; (3) achieve a high adjusted coefficient of determination ($\bar{R}^2$); and (4) get fairly good results from the diagnostic tests.

Note the following on the "restricted regression." First, the BT, a standard F-test on the lagged levels with critical values obtained from Table CI(iii), Case III of Pesaran, et al. ([31, p. 300]), rejects H_0 at the 1-percent level, as $F = 7.9$, suggesting that there does exist a levels relationship like (2). Second, there is no evidence for heteroscedasticity at the 5-percent level, as the p-value of a variable deletion test on the slope coefficient in the regression of the squared residuals on the squared fitted values is 0.09 (using robust standard errors). Third, there is no evidence for autocorrelation of order 1 to 4, since the p-values of the standard Breusch-Godfrey LM test ([19, pp. 1040-1041]) range from 0.24 to 0.54. Fourth, we reject normality at any level, since the p-value of the Bera-Jarque test is zero, but ignore this problem by invoking the central-limit theorem. Fifth, the p-values of the $RESET$ when testing the squared as well as the squared and the cubed fitted values as additional regressors are 0.30 and 0.0001, respectively, so we can reject the hypothesis of correct functional form even at the 1-percent level, suggesting that there may be omitted nonlinearities ([19, pp. 181-182]; [33, p. 1039]). Sixth, $\bar{R}^2 = 0.57$, which is relatively high, given that the regressand, $\Delta(IK_t)$, is a differenced variable ([18, p. 119]). Seventh, the estimated coefficient of Q_t in Equation (2) is positive and statistically significant at the 1-percent level (considering a one-sided alternative), namely, $\hat{\mu}_1 = 0.0142$ ($t = 2.49$).

Eighth, using a Gauss-Newton regression (GNR), we test the hypothesis of structural stability during the time periods 1963:1-2022:2, 1996:2-2001:2, 2001:3-2003:1, and 2008:1-2010:1, because of the events described earlier (when we defined the four dummies). For example, consider testing for a break in 1963:1, which is assumed to cause both the intercept and the slopes to change. Let R_t be the series of the residuals from the restricted regression and construct the interactions $X_{1t} = D63_{t-1} \times IK_{t-1}$, $X_{2t} = D63_{t-1} \times Q_{t-1}$, and $X_{3t} = D63_{t-1} \times l_{t-1}$. The GNR test is a variable deletion test on $D63_{t-1}$, X_{1t}, X_{2t}, and X_{3t} in the OLS regression (no intercept, robust standard errors) of R_t on all the regressors and on $D63_{t-1}$, X_{1t}, X_{2t}, and X_{3t}; see [28, p. 114]. The test rejects at the 5-percent level or less in all cases, except for 2001:3-2003:1 (p-value = 0.03, 0.00, 0.36, and 0.00, respectively), whereas a standard Chow test ([19, p. 232]) rejects only for 1963:1-2022:2 (p-value = 0.00, 0.85, 0.41, and 0.21). This disagreement between the two tests is not surprising. A casual look at Figure 1 reveals that the breaks in 1996:2 and 2008:1 have the form of a crash that lasts for only a few quarters, whereas that in 1963:1 has the form of a permanent increase in both the level and the volatility of IK_t (see our earlier discussion). Naturally, therefore, since the GNR test, by construction, can uncover a structural change that occurs during a short period of time, it rejects stability in 1996:2-2001:2 and 2008:1-2010:1, in addition to 1963:1-2022:2, whereas the Chow test, which tests the hypothesis of structural stability between the time period before a specific date and the period after that date, rejects only in the case of 1963:1-2022:2.

Thus, we introduce the dummies $D63_{t-1}$, $D96_{t-1}$, and $D08_{t-1}$, to allow for changes in the intercept, and the interactions X_{1t}, X_{2t}, and X_{3t}, to allow for changes in the slopes as well. The result is the "unrestricted regression" in Part B of Table 2. The fit is better now, as $\bar{R}^2 = 0.60$; the BT rejects again the hypothesis $H_0 : a_1 = a_2 = a_3 = 0$ at the 1-percent level ($F = 12.8$); there is no evidence for heteroscedasticity and autocorrelation at the 5-percent level; and the Chow test fails to reject structural stability in 1963:1 at the 5-percent level (p-value = 0.06). The $RESET$ still rejects, however, the hypothesis of correct functional form even at the 1-percent level, as

Table 2: Estimation and testing of Equation (6), with the restrictions $b_1 = b_2 = ... = b_6 = 0$ imposed (Part A) and with no restrictions (Part B)

$\hat{a}_0$	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{b}_6$	$\hat{\mu}_0$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\lambda}$
Part A. Restricted regression ($b_1 = b_2 = b_3 = b_4 = b_5 = b_6 = 0$)													
0.006	-0.075	0.001	0.03	–	–	–	–	–	–	0.078	0.014	0.403	0.635
(2.7)***	(-3.7)***	(2.2)**	(1.8)**							(6.7)***	(2.5)***	(1.4)*	(2.9)***
Bounds test and diagnostic tests													
Bounds test (H_0: $a_1 = a_2 = a_3 = 0$): $F = 7.9^{***}$; $\bar{R}^2 = 0.57$; *RESET*: $\chi_1^2 = 1.1$ [0.30], $\chi_2^2 = 18.5$ [0.0001]; Normality: $\chi_2^2 = 93.3^{***}$ [0]; Homoscedasticity: $\chi_1^2 = 2.9$ [0.09]; No autocorrelation of order 1 to 4: $\chi_1^2 = 1.4$ [0.24], $\chi_2^2 = 1.2$ [0.54], $\chi_3^2 = 3.5$ [0.32], $\chi_4^2 = 4.8$ [0.31]; Structural change: (1) GNR test (χ_4^2): 1963:1, 10.5^{**} [0.03]; 1996:2, 47.0^{***} [0]; 2001:3, 4.4 [0.36]; 2008:1, 241.4^{***} [0.00]; (2) Chow test ($F_{15, 262}$): 1963:1, 2.9^{***} [0.00]; 1996:2, 0.6 [0.85]; 2001:3, 1.0 [0.41]; 2008:1, 1.3 [0.21].													
Part B. Unrestricted regression													
0.029	-0.415	0.010	0.152	-0.012	0.003	-0.003	0.248	-0.010	-0.112	0.098	0.003	0.241	0.491
(3.5)***	(-5.1)***	(3.2)***	(2.4)***	(-1.4)	(4.3)***	(-4.8)***	(3.0)***	(-3.1)***	(-1.6)	(23.0)***	(0.9)	(2.1)**	(4.3)***
Bounds test and diagnostic tests													
Bounds test (H_0: $a_1 = a_2 = a_3 = 0$): $F = 12.8^{***}$; $\bar{R}^2 = 0.60$; *RESET*: $\chi_1^2 = 1.9$ [0.17], $\chi_2^2 = 15.3$ [0.00]; Normality: $\chi_2^2 = 72.4^{***}$ [0.00]; Homoscedasticity: $\chi_1^2 = 3.3$ [0.07]; No autocorrelation: $\chi_1^2 = 1.8$ [0.18], $\chi_2^2 = 3.3$ [0.19], $\chi_3^2 = 4.9$ [0.18], $\chi_4^2 = 5.6$ [0.23]; Chow test: 1963:1, $F_{12, 259} = 1.7^{*}$ [0.06].													

Notes: (1) ***, **, * indicate statistical significance at the 1-, 5-, and 10-percent level; (2) *t*-ratios in parentheses, *p*-values in square brackets; (3) the tests for autocorrelation are standard Breusch-Godfrey tests (Greene, 2018, pp. 1040-1041); (4) the homoscedasticity test is a *t*-test on the slope coefficient in the regression of the squared residuals on the squared fitted values; (5) the normality test is the Bera-Jarque test; (6) the *RESET* is a variable deletion test on the squared as well as on the squared and the cubed fitted values when added to Equation (6) as additional regressors (Greene, 2018, pp. 181-182); (7) we use two tests for structural change: the first is a GNR test on the coefficients of the lagged levels in the restricted regression (MacKinnon, 1992, p. 114); the second is a standard Chow test (Greene, 2018, p. 232); (8) both regressions include the following lagged differences, which are statistically significant at the 1-, 5-, or 10-percent level: ΔIK_{t-i}, i = 4-8, ΔY_{t-i}, i = 4-7, ΔQ_{t-i}, i = 3, 4.

$\chi_1^2 = 1.9$ (p-value $= 0.17$) and $\chi_2^2 = 15.3$ (p-value $= 0.0005$). In addition, the coefficient of Q_t in Equation (2) is positive but statistically insignificant, namely, $\hat{\mu}_1 = 0.003$ ($t = 0.9$). We conclude that once we take into account the two facts, (1) there are structural changes, and (2) Q_t behaves as an I(1) variable, whereas IK_t and l_t behave as I(0), so we must use the BT approach and let the coefficients change, the augmented Q_t-model seems inadequate.

Note that we recover the parameters μ_0, μ_1, and μ_2 of Equation (2) by setting all the first differences in (6) equal to zero, then leading the resulting equation by one quarter, and normalizing with respect to IK_t. Thus, for the subperiod 1947:1-1962:4, where $D63_t = D96_t = D08_t = 0$, we have that $\mu_0 = -a_0/a_1$, $\mu_1 = -a_2/a_1$, and $\mu_2 = -a_3/a_1$; whereas for 1963:1-2022:2, $\mu_0 = -(a_0 + b_1)/(a_1 + b_4)$, $\mu_1 = -(a_2 + b_5)/(a_1 + b_4)$, and $\mu_2 = -(a_3 + b_6)/(a_1 + b_4)$. Approximate standard errors of their estimators, $\hat{\mu}_0$, $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\lambda} = \sqrt{\hat{\mu}_2}$, are obtained via the "delta method" ([19, p. 229]). The coefficient estimates in Part B of Table 2 refer to the period 1963:1-2022:2. For example, $\hat{\mu}_2 = -(0.1523 - 0.1119)/(-0.4151 + 0.2475) = 0.2409 \quad (t = 2.13)$, $\hat{\lambda} = \sqrt{0.2409} \approx 0.4908$, $\quad Var(\hat{\lambda}) = (d\hat{\lambda}/d\hat{\mu}_2)^2\ Var(\hat{\mu}_2)$, hence $s.e.(\hat{\lambda}) = [1/(2\sqrt{\hat{\mu}_2})] \times s.e.(\hat{\mu}_2) = [1/(2 \times 0.4908)] \times 0.1132 \approx 0.1153$, and $t_{\hat{\lambda}} = 0.4908/0.1153 = 4.26$. For 1947:1-1962:4, we have $\hat{\mu}_2 = -0.1523/(-0.4151) = 0.3669 \quad (t = 2.26)$, $\hat{\lambda} = \sqrt{0.3669} \approx 0.6057$, and $s.e.(\hat{\lambda}) = [1/(2 \times 0.6057)] \times 0.1626 \approx 0.1342$, so that $t_{\hat{\lambda}} = 0.6057/0.1342 = 4.51$. Notice that the estimate of λ for the post-1963:1 period (0.4908) is substantially lower than that for the pre-1963:1 period (0.6057), a result that is not surprising, given the financial liberalization policies of the early 1980s.

In section 1, we mentioned that (1) the empirical failures of the Q-model applied to aggregate data are well known; (2) there have been various attempts to overcome them; and (3) to our knowledge, the issues of the possible nonstationarity of Q and the presence of structural breaks have not received proper attention. For example, Philippon ([33]) uses US aggregate quarterly data, 1953:3-2007:2, to estimate the equation in both levels and four-quarter differences without testing for unit roots and without considering structural breaks. He uses two measures of Q, the usual one (like ours) and a market-based measure constructed from corporate bond prices, which he says is "approximately stationary" (p. 1032). With the latter measure of Q his investment equation fits the post-war aggregate quarterly US data quite well, whereas the usual definition of Q fails to be significant, as in this paper. Note that when the four-quarter difference in cash flows over capital is added as an explanatory variable in his equation in differences, it turns out to be insignificant.

Andrei, et al. ([1]) also use US aggregate quarterly data, 1975-2015, to estimate the original Q-model, which excludes l_t. They, too, fail to consider the issues of nonstationarity and structural breaks. Based on the value of R^2, they write: "the aggregate investment-Q regression has worked remarkably well in recent years" (p. 252). Based on our data, we are unable to confirm this conclusion, however. First, these authors do not report unit-root tests, assuming implicitly that Q_t is I(0). Since we find that (1) Q_t behaves as I(1), and (2) the Durbin-Watson statistic (DW) for the entire sample period for their regression is only 0.23, we can conclude that the Andrei, et al. ([1]) regression may be spurious ([15]). Second, Andrei, et al. ([1]) report $R^2 = 0.065$ for the period 1975-1995 and $R^2 = 0.712$ for 1995-2015, whereas our estimation of their regression yields $R^2 = 0.139$ for 1975:1-1994:4 and $R^2 = 0.221$ for 1995:1-2022:2. Third, as we have seen, there is evidence for omitted nonlinearities, structural breaks, and insignificance of Q_t.

Next, we compare our estimate of λ for the post-1963:1 period, 0.49, which is significant at the 1-percent level, and our 95-percent confidence interval (CI), $0.26 < \lambda < 0.72$, with estimates of λ obtained from firm-level panel data reported by other researchers. First, in a survey conducted on 25 November 2008 by Campello, et al. ([7, pp. 473 and 476]), 436 chief financial officers (CFOs) of "small" public and private US firms with annual sales < \$1 b. and 133 CFOs of "large" firms with annual sales > \$1 b. were asked whether their companies are "not affected," "somewhat affected," or "very affected" by credit constraints. 41 percent of the "small" firms answered, "not affected," 37 percent "somewhat affected," and 22 percent "very affected." For the "large" firms, the corresponding percentages were 49, 35, and 16. Merging the categories "somewhat affected" (163 small + 47 large = 210 firms in a total of 569 firms) and "very affected" (94 small + 21 large = 115 firms) into one category, "financially constrained," yields $\hat{\lambda} = \frac{(210+115)}{569} = 0.57$. Using the basic formula for the standard error (s.e.) of the sample proportion, we calculate $s.e.(\hat{\lambda}) = [\hat{\lambda}(1 - \hat{\lambda})/n]^{0.5} = 0.0208$ and construct the following 95-percent CI: $0.53 < \lambda < 0.61$.

Second, using Whited's ([35, pp. 1438-1442]) firm-level data, we obtain the following two estimates of λ, which differ significantly from each other because they are based on different sample-splitting criteria. Using her classification of 206 firms (of the 325 US firms in her sample) as "constrained," because they had not received a bond rating from *Moody's* by the time of the first year of the sample period, we get $\hat{\lambda} = \frac{206}{325} = 0.63$ and the 95-percent CI $0.58 < \lambda < 0.68$. On the other hand, using her classification of 108 firms as "constrained," because they had a high debt-to-assets ratio, a group that also contained the smallest firms (p. 1440), we get $\hat{\lambda} = \frac{108}{325} = 0.33$ and the 95-percent CI $0.28 < \lambda < 0.38$, which differs significantly from the previous one, as it does not overlap with it.

Third, we obtain similar results using the firm-level data reported by Gilchrist and Himmelberg ([17, p. 556]), namely, $\hat{\lambda} = \frac{291}{428} = 0.68$ (firms without bond rating) and $\hat{\lambda} = \frac{106}{428} = 0.25$ (small size or low-dividend payouts), with 95-percent CIs $0.64 < \lambda <$

0.72 and $0.21 < \lambda < 0.29$, respectively. Again, these two CIs differ significantly from each other.

Notice that the CIs based on firm-level data are narrower than the one we constructed with aggregate data, $0.26 < \lambda < 0.72$. But since different sample-splitting criteria result in significantly different point or CI estimates of λ, there is uncertainty as to which one to adopt. This is the over-identification problem referred to in section 1.

4. Summary and conclusions

In the standard model for investment, i.e., Tobin's Q-model augmented to include a liquidity variable, l_t (cash flow or retained profits), which is usually applied to firm-level panel data, the coefficient of l_t is used as a measure of the nexus between investment and credit constraints. It lacks a precise economic meaning, however, and is econometrically over-identified, in that its estimates differ significantly, depending on which sample-splitting criterion is used to classify firms as financially constrained and unconstrained. We avoid these problems by using aggregate data. We derive our estimating equation by combining financially constrained and unconstrained firms in the same manner as in the aggregate consumption literature. Under certain assumptions, the coefficient of l_t in our equation is the square of the fraction of financially constrained firms (λ), an exactly identified parameter.

Using US aggregate quarterly data, 1947:1-2022:2, and the "bounds testing" (BT) approach, we present evidence that the standard Q-model that allows its parameters to change because of structural breaks performs poorly, as the coefficient of Q_t is not statistically significant. This finding, coupled with evidence for omitted nonlinearities, suggests that the model may be inadequate. Note that the choice of the BT approach is necessary in the present application, as Q behaves as an I(1) variable, whereas the other variables behave as I(0). In addition, allowing the coefficients to change is also necessary, as there is evidence for a structural break around 1963:1, possibly because of the investment tax credit introduced in October 1962 as well as the financial liberalization and the supply-side fiscal policies enacted in the early 1980s, particularly the 1981 and 1982 tax acts aimed to boost investment ([30]; [34, p. 5]). To our knowledge, the possible nonstationarity of Q and the presence of structural breaks have not received proper attention in the literature, and this may be the reason for some sanguine conclusions like "the aggregate investment-q regression has worked remarkably well in recent years" ([1, p. 252]). To arrive at a final verdict, however, regarding the success of the augmented Q-model for investment applied to aggregate data, one needs to apply it properly to aggregate data from other countries as well.

To the extent that our estimate of λ for the post-1963 period (0.49) can be considered reliable, we can conclude that about 49 percent of the US firms may be giving up attractive investment opportunities because of financial constraints, thus hurting economic growth. The monetary authorities might want to put in place policies that will reduce the cost of external finance, domestic and foreign, thus encouraging the implementation of efficient investment projects that would otherwise be forgone. The fiscal authorities may also want to consider instituting tax policies that can encourage investment of the financially constrained firms, e.g., reduction of the marginal corporate tax rates, subsidization of the price of investment, depreciation allowances, etc. (see [30]; [2, pp. 21-23 and 27]; [20, p. 1328]; [24, pp. 218-219]; [8, pp. 235 and 239]). As Tatom ([34, p. 5]) notes, "actions like those adopted in the early 1980s that provide generous new tax credits for investment or accelerate depreciation will hasten the replacement of obsolete plant and equipment and make possible the purchase of new facilities that otherwise might not have been considered."

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Appendix A. Construction of the series for the capital stock

To construct the series K_t, we first take the following three annual series from BEA, National Data, Fixed Assets Accounts Tables (Table 2.1), b. \$, end-of-period estimates, 1947-2020: (i) $NREQ$ = nonresidential equipment; (ii) $NRST$ = nonresidential structures; and (iii) $NRIPP$ = nonresidential intellectual property products; so our annual series is $K_t = NREQ_t + NRST_t + NRIPP_t$. We obtain the quarterly series by simply using the same annual figure for each quarter of a given year. For the time period 2021:1-2022:2, we generate the data for K_t using the identity $K_{t+1} = K_t + PGFNRI_t - CFC_t$, where CFC_t = consumption of fixed capital of private domestic businesses (BEA, Table 5.1).